

Exercise 32

Evaluate the integral.

$$\int_{\pi/4}^{\pi/3} \csc^2 \theta \, d\theta$$

Solution

Use the formula for the derivative of cotangent on page 193 to rewrite the integrand. Then use part 2 of the fundamental theorem of calculus to evaluate the integral.

$$\begin{aligned} \int_{\pi/4}^{\pi/3} \csc^2 \theta \, d\theta &= \int_{\pi/4}^{\pi/3} -\frac{d}{d\theta}(\cot \theta) \, d\theta \\ &= -\int_{\pi/4}^{\pi/3} \frac{d}{d\theta}(\cot \theta) \, d\theta \\ &= -(\cot \theta) \Big|_{\pi/4}^{\pi/3} \\ &= -\left(\cot \frac{\pi}{3} - \cot \frac{\pi}{4}\right) \\ &= -\left(\frac{1}{\sqrt{3}} - 1\right) \\ &= 1 - \frac{1}{\sqrt{3}} \\ &= \frac{\sqrt{3} - 1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{3 - \sqrt{3}}{3} \end{aligned}$$